Matrices

A matrix is simply a rectangular array (or table, like a spreadsheet) of numbers. If we want to specify what kinds of numbers our matrix contains, we might write $A \in \mathbb{R}^{m\times n}$, which means the matrix A has m rows and n columns, and each entry is a real number ($A \in \mathbb{C}^{m\times n}$ means the entries can be complex numbers, $A \in \mathbb{Z}^{m\times n}$ restricts the entries to be integers, etc.). If a matrix A has m rows and n columns, we say A is "m by n" or we write A is $m \times n$. When you see $m \times n$ it always means m rows and n columns. m is the row dimension of A and n is the column dimension, and the dimensions of A are $m \times n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

means A has m rows (each going across) and n columns (each going down), and the number in the 3^{rd} row and 4^{th} column is a_{34}. The entry in the ith row and jth column is a_{ij}.

$$A = (a_{ij})\ {}^{j=1:n}_{i=1:m}$$

says the same thing. If we know the dimensions of A already, then we might simply write

$$A = (a_{ij}).$$

There is nothing special about the symbols m and n; we could have said "A is $r \times c$" or $A \in \mathbb{R}^{r\times c}$ meaning A has r rows and c columns.

Here is a 3×4 matrix

$$A = \begin{bmatrix} 3 & 0 & 1 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 9 & 5 & 3 \end{bmatrix}.$$

The entry in the 2^{nd} row, 4^{th} column is $a_{24} = 1$, and $a_{32} = 9$, and so on. Often we will specify a matrix by it columns, like

$$A = [a_1, a_2, \ldots, a_n],$$

which means the first column of A is a_1, the second is a_2 and so on, so that the j^{th} column of A is a_j. In the example just above, $a_3 = \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix}$. It is fine to call a_3 a matrix with one column or to call a_3 a vector.

Finally, regarding notation: we don't distinguish betwen square brackets and paranthesis, so for example,

$$A = \begin{bmatrix} 3 & 0 \\ 6 & 4 \\ 4 & 9 \end{bmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 4 \\ 4 & 9 \end{pmatrix} \quad \text{and} \quad a_3 = \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix} = a_3 = \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix}.$$